1ST YEAR ENTRANCE COMPETITION EXAM– JULY 2022 SESSION

MATHEMATICS TEST

TIME ALLOWED 3h00

EXERCISE 1 **(4 points)**

Rank $x_i = 1$ is given for year 1998. Consumption is expressed in thousands of euros.

Year	1998	2000	2001	2002	2004
Rank of the year x_i	1	3	4	5	7
Consumption in thousands of euros y_i	28,5	35	52	70,5	100,5

1. Represent the points $P_i(x_i\,;\,y_i)$ in an orthogonal frame reference (we will take 1 cm as one unit in the abscissa and 1cm for 10 000 € in ordinate). **0,5pt**

2. Determine the coordinate of the mean point G ; place it in the previous mark. **0,5pt**

3. Calculate the linear correlation coefficient and conclude. **0,5pt**

4. A new adjustment in exponential form seems even better.

a. Copy and complete the following table knowing that $z = \ln y$. The results will be rounded to the nearest hundredth. **1pt**

xi	1	3	4	5	7	8
$z_i = \ln y_i$	3,35	...	...	...	...	4,94

b. Determine the reduced equation of the regression line from z *to* x obtained by the method of least squares using the calculator; this equation is of the form $z = cx + d;$ rounding the values of c and d to the nearest 10^{-2}. **1pt**

Deduce that: $y = 20{,}49\ e^{0{,}23x}$. **0,25pt**

c. Then estimate, using this new adjustment, household consumption in this city in 2007 to the nearest 100 € **0,25pt**

ESSFAR, une école de GEDUQUE SA, autorisation N°17-08298/L/MINESUP/SG/DDES/ESUP/SDA/AEO
Rue 1504, BP 35364, Yaoundé Omnisport Mfandena // Mail : contact@essfar.com
Web: www.essfar.com // Tel : 650 80 62 54 / 699 83 53 96 / 650 80 62 63 / 691 71 73 79

EXERCISE 2 **(5 points)**

Questions 1 and 2 are independent.

1) We throw two ordinary dice, we denote by X the random variable which takes as its value the absolute value of the difference of the two numbers drawn.
a) What are the different possible values of X ? **0,5pt**
b) Define the probability distribution of X. **0,75pt**
c) Calculate the expectation E(X), the variance V(X) and the standard deviation σ. **0,5pt×3=1,5pt**
2) Massangam is made up of 60% women and 40% men. 45% of women are traders as well as 20% of men. A resident of Massangam is chosen at random for a reception at the unity palace.
a) What is the probability p of choosing a trader? (male and female) **0,75pt**
b) What is the probability of having a man knowing that he is a trader. **0,75pt**
c) What is the probability of having a wife knowing that she is not a trader **0,75pt**

EXERCISE 3 **(4 points)**

Parts I and II are independent.

I. We consider in $\mathbb{C}^3$, the following system of unknowns x, y, z $(S): \begin{cases} x+y+z=2i-1 \\ xy+yz+xz=-2(1+i) \\ xyz=2 \end{cases}$

1-Show that $(a,b,c)\in\mathbb{C}^3$ is a solution of system (S)if and only if a,b and c are roots of the polynomial P of third degree whose leading coefficient is 1 that we will determine. **0,5pt**

2- Show that the equation $P(z)=0$ admits a real solution and only one that we will determine. **0,5pt**

3- Solve in $\mathbb{C}$ the equation $P(z)=0$ and deduce the solutions of the system (S) in $\mathbb{C}^3$. **1,5pt**

II. Considering the equation: $z^5-1=0$, calculate $\cos\frac{2\pi}{5}$ and $\cos\frac{4\pi}{5}$ **1,5pt**

EXERCISE 4 **(7 points)**

I- Consider the function f defined by : $\begin{cases} f(0)=0 \\ f(x)=\frac{x\ln x}{x+1}\text{ , si } x>0 \end{cases}$.

ESSFAR, une école de GEDUQUE SA, autorisation N°17-08298/L/MINESUP/SG/DDES/ESUP/SDA/AEO
Rue 1504, BP 35364, Yaoundé Omnisport Mfandena // Mail : contact@essfar.com
Web: www.essfar.com // Tel : 650 80 62 54 / 699 83 53 96 / 650 80 62 63 / 691 71 73 79

1. Study the differentiability of f at 0. **0,5pt**
2. Let the function g be defined on $]0, +\infty[$ by : $g(x) = \ln x + x + 1$.

a) Study the variations of g. **0,75pt**

b) Prove that the equation $g(x) = 0$ admits a unique solution β such that: $0,27 < \beta < 0,28$ **0,75pt**

c) Give the sign of $g(x)$ in terms of x. **0,25pt**

3) a) Express $f'(x)$ in terms of $g(x)$. **0,5pt**

b) $f(\beta) = -\beta$ **0,25pt**

c) Plot the curve f in an orthonormal frame of reference (O, I, J) of the plane. **0,5pt**

II- We define the function h defined on $]0, +\infty[$ by $h(x) = e^{\frac{x+1}{x}}$

1. Prove that the equation $f(x) = 1$ has a unique solution α in $[3,4]$. **0,25pt**
2. Show that $f(x) = 1$ if and only if $h(x) = x$. **0,25pt**
3. Show that for any x element of $[3,4]$, $h(x)$ is also an element of $[3,4]$. **0,5pt**
4. Prove that $|h'(x)| \leq \frac{1}{2}$ for any element of $[3,4]$. **0,5pt**
5. Let U the sequence be defined by $\begin{cases} U_0 = 3 \\ U_{n+1} = h(U_n), \ n \in \mathbb{N} \end{cases}$

a) Prove that for all natural numbers n, $U_n \in [3, 4]$. **0,5pt**

b) Prove that for all natural numbers n, $|U_{n+1} - \alpha| \leq \frac{1}{2}|U_n - \alpha|$. **0,5pt**

c) Deduce that for all natural numbers n, $|U_n - \alpha| \leq \left(\frac{1}{2}\right)^n$. **0,5pt**

d) Prove that the sequence U is convergent and find its limits. **0,5pt**

e) Determine the integers n for which U_n is approaching the value of α to the nearest 10^{-2}.

0,5pt